THE TEACHING OF MATHEMATICS IN THE JUNIOR HIGH SCHOOL

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The report of the National Committee on Mathematical Requirements[1] raises a number of interesting questions in view of the particular characteristics of pupils of the seventh, eighth, and ninth grades.

The success of the rapidly spreading junior high school depends upon the development of courses and methods which conform with the demands of early adolescence and which provide the type of work best adapted to the needs and abilities of a special type of pupils who because of their maturity are above the elementary types of instruction but are not yet ready for high-school work. For this reason the characteristics of this period of child-life and the aims and purposes of instruction require careful study. To one who has made such a study the special needs become readily apparent.

The formulation of a new course of study to meet these needs after they have been recognized is in many respects a much easier administrative problem than any problem of curriculum construction now confronting the school. Especially is it easier than the problem of reconstructing the courses of the senior high school where progress is retarded because of the powerful influence of tradition. It is not even necessary to think of the junior high school as a new and separate institution. The cost of new buildings and equipment may be prohibitive in many localities and may make a separate institution impossible. What is really needed is a reorganization of the subject-matter of the three grades following the sixth, the construction of additional material, and the elimination of that which should either be dropped or post-

[1] "Junior High School Mathematics." *Secondary School Circular No. 6, July, 1920.* Washington: Department of the Interior. Pp. 10.

poned to a later period. It is true that the junior high school program is essentially one of curriculum reconstruction, whether the work is carried on in an elementary school or in a high school. In either case, a recognition of pupils of this age as requiring special types of instruction is not only desirable but necessary. Junior high school pupils need a treatment different from that suitable for the children of the first six grades, and they are too young to associate with the older pupils of the high school.

In mathematics, pupils are supposed to have acquired mastery of the fundamental operations with whole numbers and with common and decimal fractions to such an extent that they are able to perform these operations with a fair degree of accuracy and rapidity. However, we cannot merely continue the arithmetic of the seventh and eighth grades, to be followed by the traditional algebra of the first year of the senior high school, or by demonstrative geometry. The traditional courses in algebra and geometry have been worked out for pupils several years older than junior high school children, and even for them these courses have been open to severe criticism. Any attempt to bring these courses in high-school mathematics without modification into the lower grades is doomed to failure. A modern course cannot be made up on the basis of tradition, conjecture, or opinion alone; material included must be defendable on the basis of definite principles, and must be organized with reference to the real needs in the life and studies of the children.

The attitude of the National Committee on Mathematical Requirements as to selection and arrangement of material can be observed in five plans intended to be suggestive to teachers "in deciding upon a course suited to their needs." As far as the work of the seventh grade is concerned, all five plans put emphasis (1) on a variety of applications of arithmetic of the type that relates to the home, to industry, to thrift, and to various school subjects; and (2) on intuitive geometry; two of the plans emphasize algebra, presumably consisting of simple formulas. In the fifth plan the Committee gives intuitive geometry the first place.

The fact cannot be overemphasized that space is both the most available and the most concrete material. A need of some

knowledge of space relations enters into everybody's life. Hence geometry, because of its usefulness and concreteness, might be made the core of the first-year course. Its purpose should be to make the pupil familiar with the common geometric forms in nature, in buildings, and in his surroundings. It should be intuitional and constructive, but not demonstrative. It should train the hand in measuring and in the use of the simple drawing instruments. It should include the fundamental geometric constructions and a considerable amount of drawing of designs and drawing to scale.

Furthermore, the applications of arithmetic which are needed to give review and drill in the operations taught in the lower grades should relate mainly to geometry because they are then within the experience of the pupils and can be understood and studied with profit.

On the other hand, most of the other applications are remote from the pupil's experience and are therefore not understood. Hence, the time spent on teaching them is not profitably employed, especially as far as training in arithmetic is concerned. The ordinary business man prefers a pupil who is thoroughly trained in the fundamentals of arithmetic and who understands the meaning of interest, percentage, and discount, to one who has studied all the applications but is not grounded in the fundamentals.

Problems in percentage, discount, and in mensuration of lines, surfaces, and solids, formulas and graphs, are met frequently in daily work and should therefore receive considerable attention. Indeed, by eliminating many of the applications for which there is no real need, time is gained to extend the pupil's experience over the whole field of secondary mathematics and to develop a high degree of skill necessary to perform the operation accurately. Space material is the most excellent material with which to accomplish these results.

So far the argument against most of the applications of arithmetic has been that they do not really enter into the life of the pupil. It is often maintained that their usefulness is so great in the later life of the pupil that they should not be dropped so readily. As a matter of fact, the variety of problems which do

arise in the ordinary affairs of daily life and which call for the use of arithmetic is not as great as is generally supposed. Recent studies tend to show that many of the topics now taught in traditional courses of seventh- and eighth-grade arithmetic might be omitted without loss to the learner, especially since they are non-essential for future use. For example, there is very little use in common practice for the solution of problems in stocks, bonds, partial payments, insurance, bank discounts, and compound interest; all of these are remote from the experiences of the pupils. Even when these problems actually arise in a bank, it is more advantageous to do computations by means of special tables and devices. It is therefore better to place the emphasis on the few really important and useful phases of arithmetic and to postpone these other applications to a later stage. Special courses may be offered to satisfy special needs, as might be the case in commercial and industrial communities in which a sufficiently large number of pupils expect to take up commercial and industrial work. Surely, there is no justification for a whole year's course in arithmetic as the first year's work in the junior high school.

If we eliminate most of the traditional seventh-grade arithmetic, the next problem which confronts us is to replace it by more suitable material. The fact has been pointed out that the main emphasis is to be placed on geometry. Moreover, practice in English and continental schools has shown that children of junior high school age can master much of the work which is now taught in the senior high school.

Recent textbooks on junior high school mathematics all show that a considerable amount of algebra, intuitive geometry, and arithmetic is now commonly included in junior high school mathematics. The report of the National Committee on Mathematical Requirements also expresses the view that arithmetic, intuitive geometry, algebra, numerical trigonometry, and demonstrative geometry form a suitable course. There is, however, difference of opinion regarding the arrangement of these subjects. As a rule, the material is divided into large topics, artificially separated from each other, frequently in such a way that none has any particular relation to the subsequent topic. Thus, geometry

sometimes follows the study of arithmetic; for example, measurement of lines is taken up after a chapter on interest.

There seems to be a feeling that mastery can be attained only by studying a single large topic for a considerable length of time. This is contrary to actual teaching experience. Lasting understanding comes with wide experience in a large number of situations, while material learned by drill and by intensive study alone of a single topic is likely to be retained only temporarily and does not transfer. We have very good evidence of this in the traditional division of senior high school mathematics, where algebra is studied for a year, then dropped, and taken up again in the third year. Invariably a review is necessary before the new work of the third-year course can be studied profitably. Most textbooks recognize the need for this review by beginning the advanced course with a summary of the first-year course. If algebra were used in connection with the second-year course, such reviews would not be necessary.

Another illustration is the well-known fact that pupils cannot apply their mathematics when needed in a different subject, as, for example, in physics even when they have done excellent work in mathematics.

The difficulty is not removed by teaching these large topics in parallel courses. It is more likely to be increased. The pupil finds himself abruptly carried from one experience into another distinctly remote from the former, with the result that he becomes confused and attains no mastery of any topic. The mathematics of the future junior high school is to be of the correlated type, which is more and more replacing traditional courses in the senior high school and even in the junior college. By giving wide experiences with work which pupils understand and which appeals to their interest, a familiarity is secured with the use of the mathematics likely to be functional in the solution of real problems. To master material which he can understand and therefore can learn has the effect of making the pupil feel and value the power of his mind.

What is needed is a complete outline of a course in which the various subjects are correlated whenever one supplements the other. Arithmetic is to be reviewed through the constant use, not by having

formal drill only. Algebra is to come in as a means of stating geometric facts and principles wherever it is needed and useful, but a scientific treatment is not to be attempted at this stage. Since the ordinary teacher is too busy to make this correlation it is the duty of those who are working on courses of study to exert their efforts mainly in this direction. Separate outlines of topics in arithmetic, algebra, or geometry, contribute but little to the solution of the problem.

The following outline indicates what was done last year toward solving this problem in one of the experimental classes of the junior high school at the University of Chicago. In view of the fact that intuitional geometry furnishes the most concrete material, it is made the means of unifying the work of this grade. Free use is made of arithmetical or algebraic material wherever it is helpful or needed, if the pupil is able to comprehend it. Occasionally, a supplementary topic is studied intensively, to the exclusion of all others, to attain mastery, as learning to solve equations of a certain type or some arithmetical operations like square roots or abbreviated multiplication. In every case only as much of a supplementary topic is studied as is needed to help the pupil to go forward with the work of the main topic.

The course is planned to have a psychological arrangement, teaching the simplest things first. Line-segments, angles, perimeters, and linear functions precede areas and quadratic functions; these in turn are followed by the study of volumes and functions of degree higher than the second.

OUTLINE OF THE SEVENTH-GRADE COURSE

I. LINE-SEGMENTS. LINEAR FUNCTIONS OF ONE VARIABLE

a) *General concepts.*—Geometric lines are met in the form of boundaries in maps, drawings, polygons, and solids. The intersections of lines are points. Points are denoted by capital letters and lines by two capital letters representing points.

b) *Measurement.*—The pupil learns to measure segments found in objects in the classroom, in scale drawings, and in graphs. He becomes acquainted with, and learns to use, the instruments for measuring, the inch ruler, the centimeter ruler, the compass,

and squared paper. He studies the metric system. He sees that measurement is approximate to a certain degree of accuracy and that the degree of precision is expressed by the number of significant figures. In measuring perimeters he needs and uses the operations with common and decimal fractions, and considerable drill in these operations is given. The names of polygons are learned, and they are classified as to sides. Various notations for numbers are given. Letters are used to denote the number of units in line-segments which have not actually been measured, leading to the idea of literal number. Some of the simplest symbols are introduced, as, $=$, $>$, $<$.

c) *Drawing.*—Facts and relations between different facts are represented in various forms, by means of tables, as temperature, train times, interest; by line or bar graphs, as statistical data; by means of an algebraic formula, as uniform motion and percentage. Some of these are represented in all three ways. The algebraic representation leads to equations, as $d = 20t$, $p = 4x$, and to the evaluation of expression of the form ax, where a is a numerical coefficient.

Many other problems, not of geometric content, are solved by means of equations of this type and are here studied, including percentage and interest problems. In this manner the pupils solve these equations in a number of different situations. No attempt is made to include equations of forms other than $ax=b$.

Scale drawings in considerable number give the pupil practice in handling instruments and include reviews of the preceding arithmetical operations. The drawing of designs aims to develop appreciation of geometrical forms.

d) *Habits of study.*—Throughout the study of any topic the main emphasis is placed on neatness, drill, and accuracy. Pupils at this age have a tendency to hurry, to be inaccurate. They must learn that speed without accuracy is not sufficient. To train neatness, the formal arrangement of all written work is illustrated by many typical examples completely worked out, to be used as patterns by the pupils. Drill is introduced wherever it is needed.

e) *Tests* are given to determine the results of the teaching, and to show pupils where further drill and practice is needed.

II. ANGLES. USE OF THE PROTRACTOR. POLYNOMIALS OF THE FIRST DEGREE

a) *General concepts.*—Angles are seen in the classroom, on models, and drawings. They occur in problems in surveying and navigation. Angles are formed by rotation of a line. Various notations for naming angles are taught.

b) *Measurement of angles with the protractor.*—The degree is used as angular unit. The sizes of angles are estimated before measuring. Angles are classified. Measuring angles in polygons leads to functional relations and to equations of the form $ax+bx+cx=d$, where a, b, c, and d are numerical coefficients.

Many problems are given which are solved by means of equations of this form, including some of the arithmetical applications. There is a considerable amount of arithmetical work in evaluating algebraic polynominals of the form $ax+bx+cx$.

c) *Drawing.*—The protractor is used to draw angles of given sizes, parallel lines, perpendicular lines, angle bisectors, and triangles from given parts. The last give the pupils the first notions of congruence.

d) *Indirect measurement.*—Having learned how to determine distances and angles by actual measurement, the next problem is to find the unknown parts by *indirect* measurement. Here we have some of the most interesting applications of geometry. Various methods are taught showing the pupil how the methods of mathematics are continually improved and opening the way for the study of future mathematics.

(1) *The congruent-triangle method.*—One or two of these problems may actually be worked out of doors. The solution calls for the construction of two congruent triangles. The pupil is led to see that this is often impracticable and even impossible and that a better method is needed.

(2) *The scale-drawing method.*—Besides the measurements, the scale-drawing method requires ruler, protractor, and squared paper for solution. It has the advantage over the congruent-triangle method that after the measurements are obtained the problem may be worked out in the classroom or at home.

(3) *The similar-triangle method.*—Here the problem is solved by means of an algebraic equation. Accuracy no longer depends upon careful construction of a figure, a rough sketch being sufficient. It shows how much more economically and accurately a problem may be worked by algebra than by geometry by one who understands the subject and furnishes a good motive for further study. The pupil learns to solve equations of the forms

$$\frac{x}{5}=\frac{8}{15},\ \frac{x}{2}+\frac{x}{5}=3,$$

and many problems are given which lead to such equations.

(4) *The trigonometric method.*—No attempt is made to define or teach trigonometric ratios. It is, however, the pupil's first experience with the use of the tangent ratio. He learns that the size of the angle determines this ratio, and that by finding the ratio he can determine the corresponding angle by means of a table. He sees the advantage of this method, as only two measurements are needed. The way is pointed to further study of the subject of trigonometry.

III. USE OF THE COMPASS. THE CIRCLE

a) *Use of the circle.*—So far the compass has been used only as an instrument for measuring. From now on it is also an instrument for drawing circles and arcs. The circle is a closed curved line, and the terms radius, diameter, arc, chord, central angle, etc., are introduced when and where they are needed.

The circle is used in gas meters, longitudes and graphs. Drawing designs is work which pupils greatly enjoy, shows geometric forms in a variety of relations, and gives excellent training in handling ruler and compass. Further use of the circle is found in the fundamental constructions of geometry, in constructing regular polygons, and triangles from given data. The protractor is used to check the accuracy of the drawings.

b) *Circumference.*—Numerous problems involving the length of a circle are worked out. They are always solved by means of the equation $c=\pi d$, another equation of the general form $ax=\mathrm{b}$. Further drill is given in the use of formulas, tabulation and graphical representation, functional relation and variation, evaluation and approximation of results.

c) Problems leading to equations of the form $ax=b$ and $ax+b=c$.—The applications which are here given are solved by equations of such forms. Considerable drill in manipulating these equations is given at this time.

IV. AREAS. FUNCTIONS OF THE SECOND DEGREE

There has been no need for functions of degree higher than the first and therefore the pupil so far has not met a quadratic function. The study of algebra is now further extended with the study of areas.

a) The rectangle.—

(1) *Properties of the rectangle.*—Some of the fundamental properties of the rectangle are found by drawing and measurement. Thus the usual theorems of geometry expressing relations of sides, angles, and diagonals are discovered and learned.

(2) *Area of the rectangle.*—The area is found first by counting the number of unit squares. Later the formula bh is worked out and used. Many opportunities for correlation arise. Substituting particular values in the formula gives review of products of whole numbers, and common and decimal fractions. Combination of several terms of the form bh forms problems of evaluating functions like $\frac{ab+ac+de}{a+bd}$. Keeping the base fixed and varying the altitude brings in graphical representation and variation.

The parenthesis is used for the first time in finding the area of a rectangle of dimensions $a+b$ and c, and $a+b$ and $c+d$. This is made the basis for multiplying polynominals by monomials, and polynominals by polynominals, but all resulting products are either of the first or of the second degree only.

The study of equations is further extended to the form $3(x+5)=22+x$.

b) Area of the square.—As in the rectangle the area is found by counting units and by formula. The following algebraic topics are taught: $(a+b)^2=a^2+2ab+b^2$; the equation $a^2=144$; graph of the function $A=a^2$, the first graph which is not a straight line; square root; the theorem of Pythagoras and equations of the form $x^2+a^2=b^2$.

c) *Areas of the parallelogram, trapezoid, triangle, and the circle.*—In each case the pupil has new experiences with the mathematics already learned, and some extensions are made. Generally speaking, the principal topics are computation, evaluation, drawing, measuring, graphing, study of the formula, and of the equation.

The outline given so far is sufficiently detailed to show that the various subjects of mathematics can be correlated in a natural and simple manner. The remainder of the course takes up the following topics.

V. SURFACES. VOLUMES. FUNCTIONS OF DEGREE HIGHER THAN THE SECOND

No factoring is needed so far and therefore factoring is not taught. Similarly, there has been no need for simultaneous linear equations, positive and negative numbers, and quadratic equations of the form $ax^2+bx+c=0$. These topics are therefore all postponed to a later stage. There has been a considerable amount of informal reasoning, but no demonstrative geometry.

In conclusion, the example below serves as a cross-section, showing to what extent a correlation exists. The following topics have been studied in connection with the rectangle:

Quadrilaterals, perpendicular and parallel lines, equality of sides and angles, sum of angles, equations of the form $ax+bx+cx+dx=360$.

Perimeters, evaluations of $2x+3y$, solution of $3x+x=60$.

Diagonals, congruent triangles,equations of the form $a^2+b^2=c^2$.

Parallel lines, alternate interior angles, angular relations.

Areas, squared paper, approximation, variation, tables, graphs, formulas, multiplication of polynomials.

The operations with arithmetical numbers, and applications of arithmetic.

Throughout the time in which the course outlined above was worked out careful analytical tests were given. The results have shown that this type of work can be taught with a high degree of attainment. The great amount of concrete material undoubtedly increased clear understanding, and experiences in so many different situations brought about results at least as good as are commonly attained by intensive drill, but more lasting.